Experimental Modal Analysis of Two-wheeled Vehicles. Prediction of the response to road unevenness

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Keywords: comfort, motorcycle, modal analysis

SUMMARY: The experimental study of the vibrations of two-wheeled vehicles gives useful results when it is carried out exciting the whole vehicle through the wheels. Modal analysis techniques make it possible to identify both in-plane modes, which influence comfort, and out-of-plane modes, which influence stability and handling. In the paper some results in terms of natural frequencies, damping coefficients and modal shapes are presented. The possibility of predicting the comfort on the road making use of the transfer functions measured in laboratory test is highlighted.

1. INTRODUCTION

Recently a new testing equipment for the study of motorcycle and scooter vibrations has been developed at the Department of Mechanical Engineering – Padova University. The main component of the equipment is an hydraulic shaker, which is able to carry out sinusoidal and sweep excitation of the whole vehicle in the low frequency range. Both in-plane excitation and out-of-plane excitation can be performed.

Several two-wheeled vehicles have been tested, the results presented in the paper refer to a super-sport motorcycle (1000 cc) equipped with a box section aluminium frame and a double-sided supported swingarm.

The first part of the paper deals with the study of in-plane vibrations and it is aimed to predict the response of the vehicle to road unevenness by means of laboratory tests. The band of frequency of excitation ranges from 1 to 20 Hz. This band is called the "ride range" and it is the most important from the comfort point of view, because the human sensitivity to whole-body vibrations and to arm-hand vibrations reaches the largest values (Griffin 2004). Moreover, the excitation generated by road unevenness shows large amplitudes in this band of frequencies, if typical road profiles (Hunt 1991) are considered and the speed of travel ranges from 5 to 45 m/s .

In the "ride range" there are the "rigid" modes of the vehicle in which the chassis, fork and swingarm behave essentially as rigid bodies, whereas the deformability is concentrated in the tires and suspensions. Near the upper limit of this band, modes of vibration that show some deformability of the structural elements may be present.

The Frequency Response Functions (FRFs) in some significant points (saddle, handle-bars) are measured both in the presence of front excitation (on the front wheel) and rear excitation (on the rear wheel). The comfort on the road is predicted combining the FRFs measured with front and rear excitation in order to take into account the wheel-base effect. This phenomenon takes place because on the road the front and the rear wheel encounter the same road disturbance with a time delay that depends on the speed and on the wheel-base.

The second part of the paper deals with out-of-plane excitation and it is aimed to highlight the presence of structural modes (dominated by the deformation of the chassis, fork or swingarm) that may affect the stability and handling properties of the vehicle.

The frequency of excitation ranges from 1 to 30 Hz. 159 FRFs are measured in a mesh of testing points in order to identify the modes of vibration. A detailed analysis of the identified modes is carried out to find the zones of the motorcycle that show structural deformation. The particular problems which are present in carrying out modal analysis of systems of bodies connected by kinematic pairs (e.g. the steer) and by springs and dampers are discussed.

2 TESTING EQUIPMENT

In the experimental tests on two wheeled vehicles it is important to direct the attention towards low frequency bands both for in-plane and out-of-plane vibrations. The principal component of the new testing equipment is an hydraulic shaker which is able to operate in low frequency range. It is controlled in closed loop by means of a Moog servo-valve and a displacement transducer (LVDT) which measures the instantaneous position of the stem.

Figure 1: experimental apparatus for in-plane testing

A NI board generates the input, which may be either a sinusoidal function or a frequency sweep with a constant amplitude. In the framework of this research a sweep function with maximum amplitude (displacement) of 2 mm and duration of 120 s is used in most cases. The range of frequency, is different for the two kind of tests: 1-20 Hz for the in-plane excitation and 1-30 Hz for the out-of-plane excitation. Actually, in the last case it is important to search the modes with structural deformation, hence, a larger range of frequencies has to be inspected.

A particular structure has been developed to guarantee the stability of the vehicle under testing without modifying the modal characteristics.

When the in-plane analysis is performed the hydraulic shaker is mounted in the vertical direction to simulate road excitation, figure 1 shows the motorcycle, shaker and structure. The motorcycle is supported in such a way to let free the suspensions' travels and the deformations of the tires. The stability is guaranteed by means of two horizontal booms, which are fixed to the structure and are in sliding contact with the tank of the motorcycle. One wheel is laid on a table mounted on the stem of the hydraulic cylinder. The other wheel is laid on a fixed table. The weight of the vehicle is completely supported by the tables. Both tables are covered with high friction material.

In the out-of-plane tests the hydraulic shaker is mounted in the horizontal direction and the stem moves a vibrating table covered with a rough surface to simulate road grip. A wheel is laid on the vibrating table, the other on a fixed table, see figure 2.

Figure 2: experimental apparatus for out-of-plane testing

For the out-of-plane tests, the motorcycle is suspended from the structure in two points, the first near the saddle and the second on the steering head. The forces exerted by the suspension system on the two points are measured by means of scales. The measurement of the forces exerted by the suspension system makes it possible the calculations of the loads acting on the wheels. Two tackles are used for calibrating the suspension forces and, hence, the loads on the wheels, assuring the repeatibility and the accuracy of the measurements. The load condition has to simulate the behaviour on the road, but at the same time, the suspension forces are needed to guarantee the stability. After some tests, for this kind of vehicle, the best testing condition was reached with a load of 620 N on the front wheel and 630 N on the rear wheel.

Two tri-axial piezo-electric accelerometers and a multi-channel spectrum analyser (STAC) are used to measure accelerations (the frequency range of accelerometers is 0.5-3000 Hz). During the tests, the first accelerometer is fixed to the stem of the actuator and the second is displaced to different points of the motorcycle that define the mesh for the vehicle under testing. The FRFs are calculated by the spectrum analyser and are defined as:

$$H(\omega)=\frac{G_{xy}(\omega)}{G_{xx}(\omega)} \tag{1}$$

in which $G_{xy}(\omega)$ is the cross spectrum between the acceleration of the mesh point and the acceleration of the stem of the cylinder and $G_{xx}(\omega)$ is the auto-spectrum of the acceleration of the stem of the cylinder .

Modal analysis is carried out by means of the ICATS code.

2. IN-PLANE EXCITATION

2.1. Prediction of comfort

Vibrations caused by road unevenness are transmitted to the rider and passenger through the saddle, handle-bars and foot-rests. Hence, the FRFs between the vertical and longitudinal accelerations in these points and the vertical acceleration of the vibrating table are measured. The weight of the rider causes large deformations of the saddle surface, which are not easy predictable. Therefore, to obtain more general results, the accelerometer is located on the connections between the frame and the saddle. For safety reasons the tests are carried out without the rider, to take into account this condition, the preload of the suspensions is set to the minimum value. The damping of the suspensions is set to the minimum value as well. The front and rear brakes are locked.

Figures 3, 4 and 5 show the FRFs measured in the vertical direction in the characteristic points (on the right side of the motorcycle) in the presence of front and rear excitation. The FRFs show the first peak or a large increase in amplitude at about 4 Hz. These characteristics may be related to the excitation of the pitch mode, which in common motorcycles, is dominated by suspensions' travels (Cossalter 2002). Above 4 Hz the FRFs with rear excitation are still large, or show a second peak at about 6 Hz (point on the foot rest); the FRFs with front excitation show a peak at about 6 Hz.

The presence of some resonance peaks in the range 4-8 Hz suggests that in this motorcycle the pitch mode may be split into two modes: a first mode at lower frequency (about 4 Hz) that involves mainly the travel of the front suspension and a mode at higher frequency that involves mainly the travel of the rear suspension.

The resonance peaks, which appear at about 11 and 13 Hz with front and rear excitation respectively, may be related to the front and rear hop modes, which are dominated by the radial deformation of tires.

Figure 6 shows the FRFs measured in the saddle in the horizontal direction. There is a large resonance peak at 4 Hz, which is related to the excitation of the pitch mode. At higher frequencies the amplitudes are much smaller than in the vertical direction.

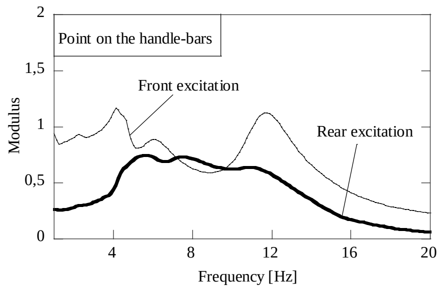

Figure 3: modulus of the FRF measured on the handle-bars, vertical direction

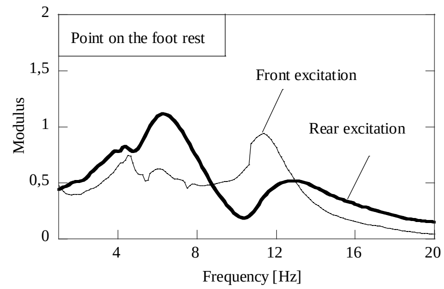

Figure 4: modulus of the FRF measured on the foot rest, vertical direction

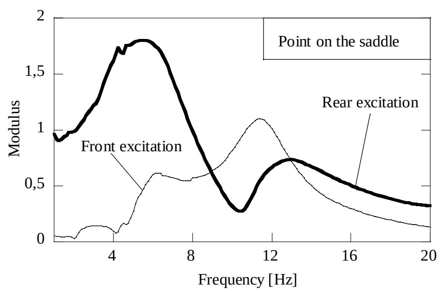

Figure 5: modulus of the FRF measured on the saddle, vertical direction

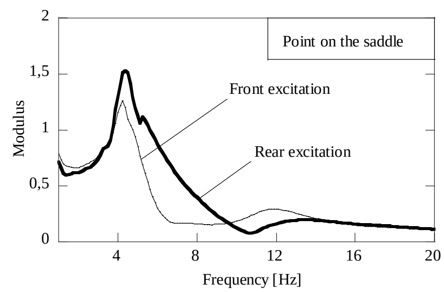

Figure 6: modulus of the FRF measured on the saddle, horizontal direction

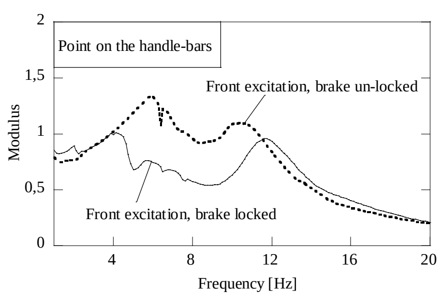

Figure 7: modulus of the FRF measured on the handle-bars, vertical direction

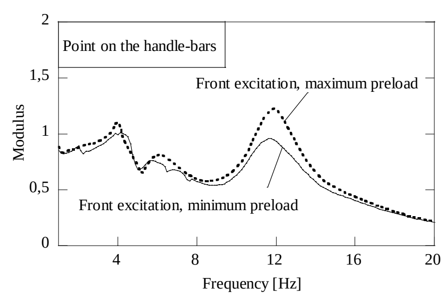

Figure 8: modulus of the FRF measured on the handle-bars, vertical direction

The testing condition have a large influence on the FRFs. Figure 7, which deals with the vertical acceleration of a point on the left side of the handle-bars, shows that if the brakes are not locked the FRF has a very different behaviour in the range 4-12 Hz. Figure 8 deals with the same point. It shows that, if the trim of the motorcycle changes because the pre-load of the front suspension increases, the FRF has larger amplitudes expecially in the range of the hop mode.

The measurements in the same point of the FRFs caused by the rear and front excitation are needed to predict the comfort of the vehicle on the road, because the same disturbance excites first the front wheel and then the rear wheel, with a time delay p/V, in which p is the wheel-base and V the speed of travel. The FRF between the acceleration of a characteristic point and the road excitation can be predicted by combining the complex FRFs measured in laboratory in the presence of front and rear excitation by means of the equation:

$$H'(\omega,V) = H_{front}(\omega) + H_{rear}(\omega)e^{-i\omega\frac{p}{V}} \qquad (2)$$

in which ω is the angular frequency. These FRFs are named correlated FRFs.

The correlated FRFs that take into account the wheel-base effect are represented in figure 9, 10 and 11. They depend on the forward speed and show series of periodic peaks, which are more evident when V is small. The frequency interval between two consecutive peaks is about V/p.

The FRFs between the accelerations in the characteristic points and the amplitude of road disturbances in meters can be calculated multiplying the FRFs of figures 9, 10 and 11 by ω^2.

Road unevenness is a random process, which can be described by means of the power spectral density (PSD) of disturbances as a function of wave-number $k = 2\pi/\lambda$, in which λ is the wavelength of disturbances. A typical expression (see Hunt 1991) of road PSD is:

$$S_{rr}(k) = S_0\left(\frac{1}{k}\right)^n \qquad \begin{cases} n = 2 & k \le 1 \\ n = 1.5 & k > 1 \end{cases} \qquad (3)$$

in which s_0 is a constant that depends on the road quality ($s_0 = 64 \cdot 10^{-6}$ $\left[m^2/rad/m\right]$ for medium quality roads).

If the vehicle moves with constant speed V the PSD in the wave-number domain is transformed into a PSD in the angular frequency domain by this equation (see Davis 2004):

$$S_{rr}(\omega,V) = \frac{S_{rr}(k = \frac{\omega}{V})}{V} \qquad (4)$$

The response to road unevenness of a vehicle running at constant speed can be described by means of the PSD of accelerations in some characteristic points.

The PSDs of accelerations in the vertical and horizontal direction are related to the correlated FRFs and to the road PSD by the following equations (see Davis 2001):

$$S_{xx}(\omega,V) = \frac{\omega^4}{V}\left|H'_x(\omega)\right|^2 S_{rr}(k = \frac{\omega}{V}) \qquad (5)$$

$$S_{zz}(\omega,V) = \frac{\omega^4}{V}\left|H'_z(\omega)\right|^2 S_{rr}(k = \frac{\omega}{V}) \qquad (6)$$

Figure 12 shows the PSD of the vertical accelerations in a point on the right side of the handle-bars of the motorcycle moving at constant speed on a medium quality road.

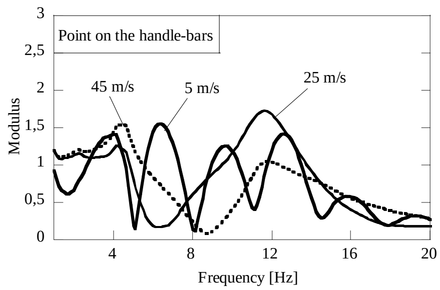

Figure 9: modulus of the predicted FRF on the handle-bars, vertical direction

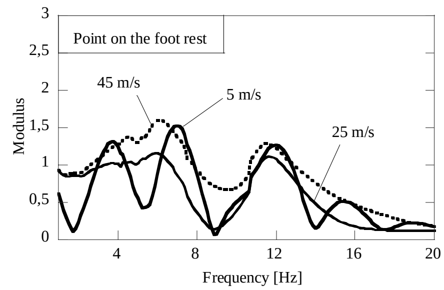

Figure 10: modulus of the predicted FRF on the foot rest, vertical direction

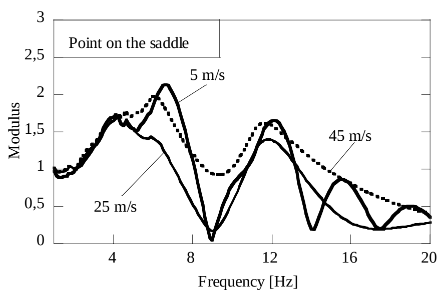

Figure 11: modulus of the predicted FRF on the saddle, vertical direction

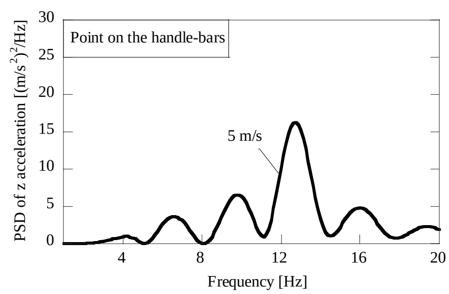

Figure 12: modulus of the predicted PSD on the handle-bars, vertical direction

2.2. Modal analysis

Figure 13 shows the first in-plane mode of the motorcycle that has been identified by means of the ICATS code. The mesh in reference conditions is represented by the grey solid line, the modal shape is represented by the black solid line. The natural frequency of the first mode is 4.4 Hz. It is a pitch mode with a relevant travel of the front fork, the points on the handle-bars show large motions both in the vertical and in the horizontal direction.

Figure 14 shows the second mode of the motorcycle, whose frequency is 5.8 Hz. It is a pitch mode as well, but in this case the motion of the rear suspension is relevant. Finally, figure 15 shows the third mode, which is a hop mode with natural frequency 11.8 Hz.

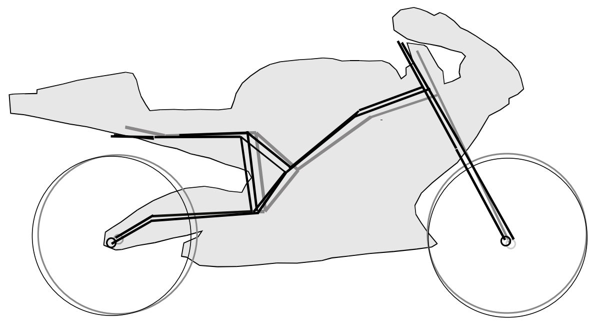

Figure 13: in-plane mode of the motorcycle at 4.4 Hz.

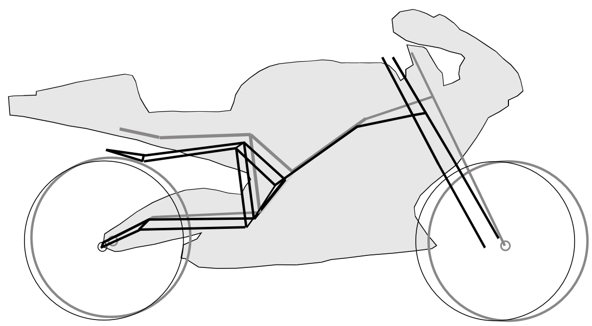

Figure 14: in-plane mode of the motorcycle at 5.8 Hz.

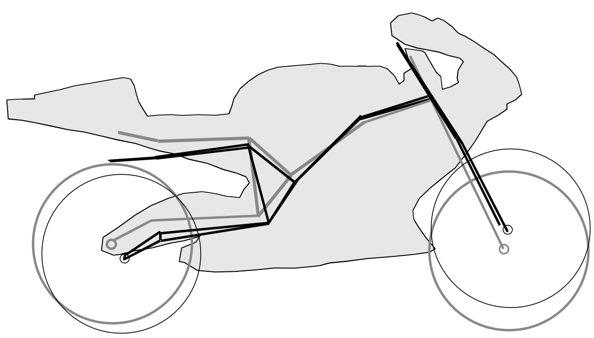

Figure 15: in-plane mode of the motorcycle at 11.8 Hz.

4 OUT-OF-PLANE EXCITATION

4.1 Base excitation

Before showing the results, some theory is useful for explaining the equivalence between the classical way for determining the FRFs and the method used in this research.

If the simple lumped-element system of figure 16 is considered, the matrix equation that gives the response to a harmonic force exerted on the first mass is:

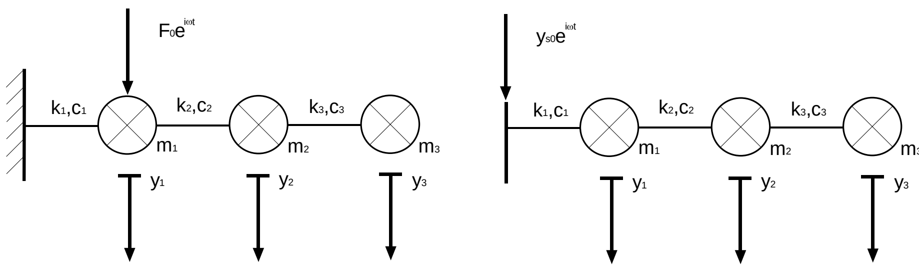

Figure 16: systems of three masses with visco-elastic connections

$$\{y\} = \left[-\omega^2 [m] + i\omega[c] + [k]\right]^{-1} \{F_0\} \tag{7}$$

in which $[m]$, $[c]$ and $[k]$ are the mass, damping and stiffness matrices respectively.

The FRFs that can be evaluated are the receptances $y_{i0}/F_0 = H_{i1}(\omega)$ in which y_{i0} and F_0 are obtainable from the experimental measurements.

If the connections between the masses and the frame are not modified and the system is excited by moving harmonically the base, the following equation holds:

$$\{y\} = \left[-\omega^2 [m] + i\omega[c] + [k]\right]^{-1} \{(k_1 + c_1\omega)\, y_{s0}\} \tag{8}$$

During the resolution of the system the role of F_0 is taken by $(k_1 + c_1 i\omega)\, y_{s0}$. The FRFs, that can be evaluated, are now ratios between displacements $y_{i0}/y_{s0} = H_{i1}(\omega)(k_1 + c_1 i\omega)$, which are equivalent to ratios between accelerations $\ddot{y}_{i0}/\ddot{y}_{s0}$, since the excitation is harmonic. $\ddot{y}_{i0}$ and $\ddot{y}_{s0}$ are obtainable from the experimental measurements. The term $(k_1 + c_1 i\omega)$ is a complex function of ω, if $c_1 i\omega$ is not negligible with respect to k_1.

In the case of the motorcycle k_1 and c_1 are the lateral stiffness and damping coefficient of the tire excited by the vibrating table and they can be obtained from independent experimental measurements (see Doria 2002). The order of magnitude of k_1 and c_1 implies that for frequencies less than 50 Hz the value of $(k_1 + c_1 i\omega)$ is a real number close to k_1. From the ratio y_{i0}/y_{s0} the term $H_{i1}(\omega)k_1$ is obtained, which is a function with different amplitude in comparison to $H_{i1}(\omega)$ but with the same form and the same damping coefficients. Therefore, the modal analysis is not distorted by the use of y_{i0}/y_{s0}.

4.2 Out-of-plane Modal Analysis

The aim of this analysis is to determine the frequency-band in which the structural deformations appear for each component of the motorcycle; figure 17 shows the location of all the points defining the mesh of the vehicle in the un-deformed condition, the direct point is placed on the rear wheel rim (point 100).

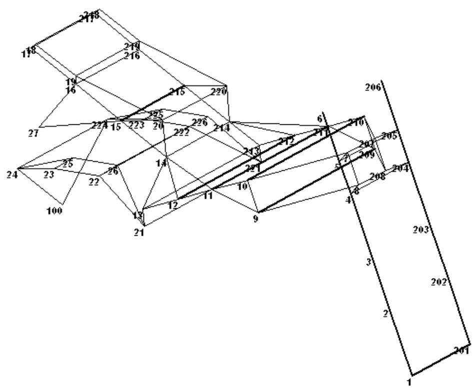

Figure 17: mesh of the whole vehicle

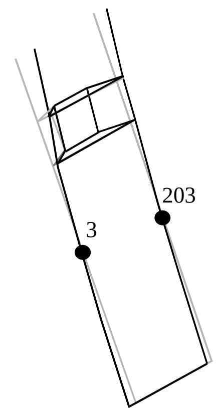

Figure 18: front fork mode at 14.0 Hz

If only the global analysis of all the acquired FRFs is carried out, it is difficult to identify the structural deformations, because a large rigid rotation of the front fork is superimposed. To avoid this problem, the motorcycle is considered as the union of three principal subsystems (the front fork, frame and swingarm) and the modal analysis is carried out for each one of them. To highlight the shape of the single mode, sometimes it is useful to consider the displacements only in the y-direction, reducing in this way the influence of the rigid components of motion.

For the front fork, the first mode which shows structural deformation is at frequency of 14.0 Hz and it is a bending of the fork's brace in y-direction with nodal points corresponding to the end of the fork-sleeves (points 3 and 203). Figure 18 shows the mesh of the front fork with this mode overlapped.

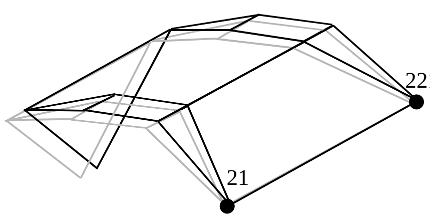

Figure 19: swingarm mode at 26.6 Hz

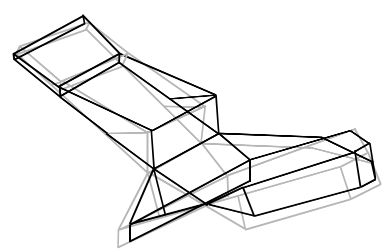

Figure 20: chassis mode at 15.6 Hz

For the swingarm, the first mode with structural deformations appears at higher frequency: 26.6 Hz. It shows a bending of the swingarm's brace in y-direction, with two nodal points (21 and 221) located at the connection points with the chassis, as can be seen in figure 19.

For the chassis, the mode at 15.6 Hz shows a torsion deformation which is more evident in the zone with the two seats; the rigid rotation of the whole sub-system around its center is present as well. Figure 20 shows the identified mode shape.

5 CONCLUSIONS

The possibility of predicting by means of laboratory tests the response of a two-wheeled vehicle to road unevenness has been studied. The measured FRFs are influenced by the testing conditions. If the proper testing conditions are found, the measured FRFs show modes of vibration that are very similar to the modes of the moving motorcycle predicted by multi-body codes. A further improvement of the research will be the test of the motorcycle with the rider.

The out-of-plane modal analysis has been carried out with the purpose of finding the frequencies of the first modes of vibration characterized by the deformation of the structural elements. The results show that some deformations of the front forks appear above 14.0 Hz. The modes of vibration with significant deformation of the chassis and swingarm take place above 15.5 and 26.6 Hz respectively.

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Acknowledgement

The authors wish to acknowledge the University of Padova for the founds to the research project 2003 "*Analisi sperimentale delle vibrazioni nei motocicli con particolare riferimento alle problematiche di comfort e sicurezza*".