An advanced multibody model for the analysis of motorcycle dynamics

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Abstract: This work presents the latest improvement of the non-linear and fully parametric motorcycle multibody software developed at DIMEG in several years. The model includes both rigid and flexible bodies, different kind of suspension linkages (such as paralever and swingarm for the rear frame, telescopic fork, telelever, and duolever for the front), a special model for the road-tire interaction and a transmission model which includes the compliance as well. The mathematical model has been entirely derived in a symbolic form, then it was implemented in a Fortran program named *FastBike*. The developed software makes it possible to carry out three different kind of simulations: steady state simulations, stability and frequency domain simulations and time domain simulations. It is worth highlighting that, due to the intrinsic instability of two wheeled vehicles, time simulations cannot be performed in open loop. Therefore, a virtual rider which includes a vehicle tracking with look ahead strategy and a self-tuning strategy has been developed. Some simulation examples are illustrated and discussed, and comparison between experimental tests are given.

Keywords: two-wheeled vehicle, multibody, tire, rider

1 Introduction

The utilization of multibody tools in motorcycle design is continuously growing and is becoming as widespread as in the other branches of automotive engineering. This work presents the latest improvement of a motorcycle non-linear and fully parametric multibody model developed in several years. The mathematical model has been entirely derived in a symbolic form by using MBSymba [1], which is a Maple package for the symbolic modelling of multibody systems developed at DIMEG, then it was implemented in a Fortran program named FastBike, [2-7]. The developed software makes it possible to carry out three different kind of simulations: steady state simulations, stability and frequency domain simulations and time domain simulations. The steady state analysis, which is not commonly available on commercial multi-body software, makes it possible to easily calculate the trim of the motorcycle in static condition, during steady cornering, while braking or accelerating, without the need of modelling rider behaviour. The modal analysis consists of the calculation of natural frequencies and damping ratios as well as Frequency Response Functions (FRF) of the linearized model in straight running, during steady cornering and while braking or accelerating. Finally, several typical manoeuvres such as slalom, lane change, entering a curve, braking may be simulated in the time domain. It is worth highlighting that, due to the intrinsic instability of two wheeled vehicles, time simulations cannot be performed in open loop. Therefore, a virtual rider which includes a vehicle tracking with look ahead strategy and self-tuning strategy has been developed [4,8].

The modelling of motorcycle dynamics has experienced many advancements in the last 40 years. A survey of the motorcycle scientific literature is reported in [9], where the well known capsize, weave and wobble vibration modes are discussed together with the steering response. Significant works on the effect of frame compliance are [10-13], whereas the passive rider mobility is addressed in [14] and the passenger effect in [15]. In [16,17] the rider’s impedance has been considered and its effect on stability discussed. The current work incorporates all the features which have shown to be significant for motorcycle modelling. In section 2 the main feature of the motorcycle and main modelling aspects are illustrated, whereas in section 3 some simulation examples are discussed and compared with test results.

2 Multibody Model Description

2.1 Equations of motion

The motorcycle dynamics is described with a moving reference frame which captures the vehicle gross motion in the road plane, i.e. the longitudinal velocity U_X, the lateral velocity U_Y and the yaw rate Ω_Z. It should be noted that the use of a moving frame is pretty common in vehicle dynamics because it has several advantages, see e.g. [18,19].

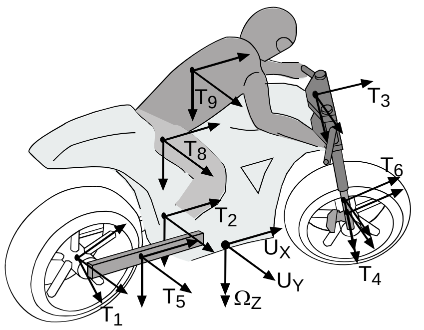

Figure 1. The motorcycle body frames.

In addition to the moving reference frame, there are nine frames attached to each of the nine bodies which constitute the motorcycle model. More in detail (see Figure 1), the frame T_1 is attached to the rear wheel, frame T_2 to the rear chassis, frame T_3 to the front frame, frame T_4 to the front wheel, frame T_5 to the non–rotating rear unsprung mass, frame T_6 to the non-rotating front unsprung mass, frame T_7 to the front chassis, frame T_8 to the lower part of the rider's body and frame T_9 to the upper part of the rider's body. The position and orientation of a generic body frame is fully identified by the coordinates of the frame origin (x, y, z) with respect to the moving axes system and the yaw (ψ), roll (ϕ) and pitch (μ) angles (SAE convention).

The equation of motion in moving frame are derived with the Newton-Euler approach for each of the nine bodies in the following form:

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}} &= \mathbf{f}(\mathbf{v},\mathbf{w},\mathbf{R},\mathbf{u}) \\ \dot{\mathbf{p}} &= \mathbf{g}(\mathbf{p},\mathbf{v},\mathbf{w}) \end{aligned} \tag{1}$$

where **M** is the (constant) mass matrix, **v** is the vector of the body velocities (v_{GX}, v_{GY}, v_{GZ}, ω_X, ω_Y, ω_Z), **w** is the vector of the moving frame velocities (U_X, U_Y, Ω_Z), **R** is the vector of reactive forces and torques, and **u** is the vector of inputs (i.e. the steering torque, engine torque and rear and front brake torques), and **p** is the position vector (x, y, z, ψ, ϕ, μ).

The constraints among the nine bodies (e.g. the steering axis revolute joints, the telescopic fork prismatic joint, etc.) give rise to a set of algebraic equations

$$\mathbf{\Phi}(\mathbf{p}) = \mathbf{0} \tag{2}$$

which generate a DAE system [20] of equation of index 3 when assembled with (1).

Since the transmission flexibility may have significant effects on traction performance and, for racing motorcycles, chatter instability [21,22], an additional degree of freedom is introduced to take into account the sprocket absorber deflection (between the rear rim and the chain sprocket), and the correspondent equation is added to (1) and (2).

The road-tire contact forces are computed according the well know *Magic Formula* for motorcycle, and two approaches are available for the force coupling: the *Similarity Method* and the *Loss Functions Method* [18]. The tire forces are applied on the actual contact point: its position on the carcass is defined by means of the tire camber α and tire slope β. Also the carcass compliance and damping is considered by means of the lateral ζ_L, radial ζ_R, and the tangential ξ deflections. This approach automatically includes the tire lag, i.e. no additional relaxation equations are necessary [5,23]. Summarizing, the tire model has five additional variables $\boldsymbol{\chi}$=(α, β, ζ_L, ζ_R ξ) and as many equations

$$\mathrm{T}(\mathbf{p},\mathbf{v},\mathbf{w},\boldsymbol{\chi},\dot{\boldsymbol{\chi}}) = \mathbf{0} \tag{3}$$

2.2 Flexible Joints & Suspension Schemes

The vehicle flexibility is modelled by means of several lumped compliances. The structural compliance is modelled by defining a Centre of Compliance (CC) and the principal directions of compliance *u*, *v*. Due to a smart approach to constraints formulation, every rigid joint of the model may be easily turned into a compliant joint. For example, the rigid steering head revolute joint restraints two (bending and pitch) out of three rotations between the front frame and the chassis, thus leaving the steer degree of freedom. In this model the torsion and pitch restraints may be relaxed, i.e. non-null values of compliance may be used. The same may be done for all other joints.

Five different suspension schemes are available all provided with their own additional lumped compliance points: the swingarm (Figure 2a) and 4-bar linkage (Figure 2b) for the rear linkage, duolever (Figure 3a), telescopic fork (Figure 3b) and telelever (Figure 3c) for the front. It has been assumed that the inertia of the linking bodies (e.g. the rockers of the four-bar linkage) is negligible. Therefore each suspension restraints the relative motion between the rear unsprung and the rear chassis in the case of rear suspensions and among the front chassis, the front frame and the front unsprung in the case of front suspensions.

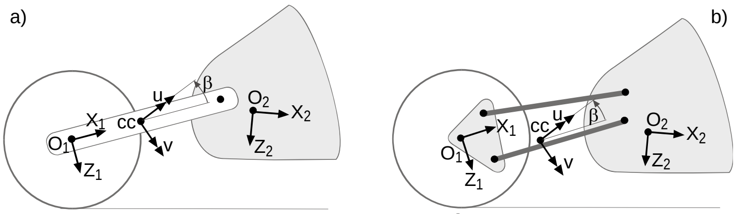

Figure 2. Rear Suspension schemes.

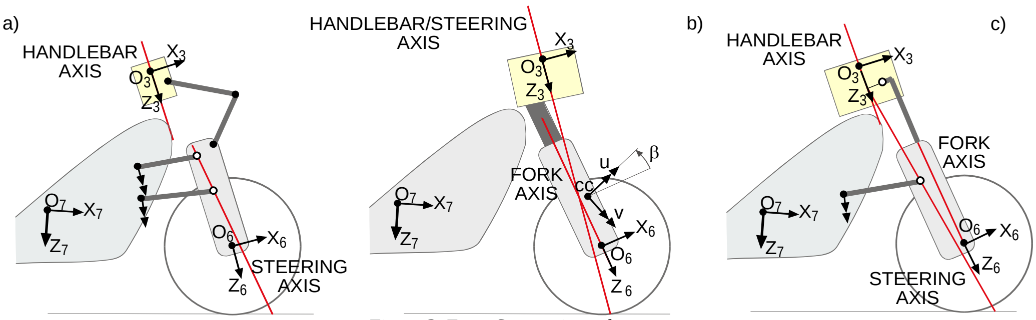

Figure 3. Front Suspension schemes.

3 Simulations

3.1 Steady State Analysis

The computation of vehicle steady-state configuration, i.e. vehicle trim given the vehicle speed and lateral acceleration, is a very important feature for parametric analyses. Many handling indexes (e.g. the acceleration index, roll index, steering ratio, etc.) may be computed without running any time simulation, thus saving a lot of time since the steady state problem requires the solution of a set of non-linear algebraic (all derivates are null according to steady state definition) equations instead of the integration of the differential-algebraic set of equations.

As an example, Figure 4 shows the comparison between the experimental and computed acceleration indexes of a naked motorcycle as a function of speed for three different cornering radii. The acceleration index is defined as the ratio between the actual steering torque and the lateral acceleration [24] and it is usually computed by means of steady turning test. For a good feeling little torque should be applied to the handlebar and preferably away from the curve. In Figure 4 the correlation between experimental and model results is quite good: given the cornering radius, the steering torque increases with speed and changes sign above a certain speed, i.e. above a certain lateral acceleration.

It should be noted that many commercial multibody codes do not allow the computation of the steady state solution, therefore closed-loop time domain simulations are necessary for the evaluation of the acceleration index. Anyway they require more time plus a control system which stabilizes the intrinsic vehicle instability.

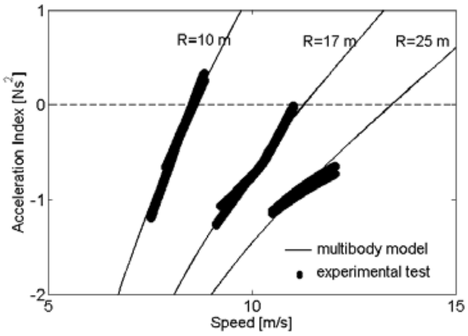

Figure 4. Acceleration index of a naked motorcycle performing a clockwise cornering.

3.2 Frequency Domain Analysis

Frequency domain analysis is an efficient approach for the evaluation of vehicle stability and the estimation of some handling indexes, since it does not require any time domain simulation. More in detail, first the equilibrium trim at a certain speed is computed by solving the plant equations in steady state condition, then the plant is linearized about this configuration and the state space matrices **A**, **B** and **C** are found so that

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{4}$$

with **x** the vector of state variables, **u** the vector of inputs (see section 2.1) and **y** vector of the observed variables. At this point the eigenvalues of **A** are computed (i.e. the stability properties are found) and the FRF is computed as

$$\frac{\mathbf{y}}{\mathbf{u}} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \tag{5}$$

where s is the Laplace variable and **I** the identity matrix.

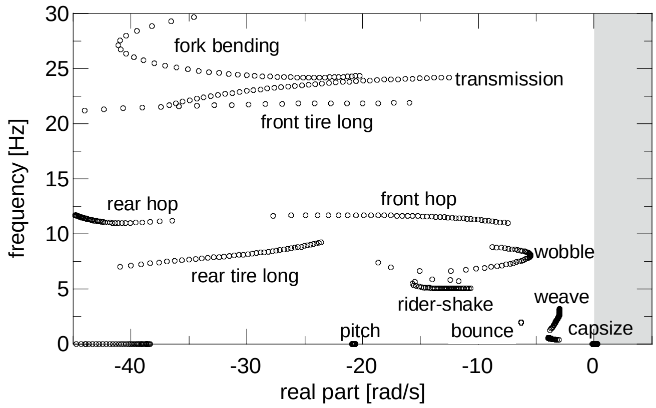

Figure 5. Root-locus for speeds from 10 to 50 m/s of a naked motorcycle.

The computation of motorcycle vibration modes is effective since it is well known [24] the connection between certain vibration modes and particular behaviour of the vehicle. More in detail, the stability is mainly related to *weave* (which is an oscillation of the whole vehicle, weave exactly) and *wobble* (which mainly consists of a steer rotation) modes: in particular the weave may become unstable at high speed (or at least it is not much damped), whereas the wobble may become unstable in the mid-speed range. The rider mobility (lateral and roll vibration) and the front *fork bending* structural mode have significant effect on their stability, see e.g. [13]. The riding comfort is mainly related to *bounce* and *pitch* modes (which involve suspensions), whereas the road holding depends on *front hop* an *rear hop* modes (which involve the tire radial deflections). *Transmission* mode (related to the deflection of the sprocket absorber on the rear wheel) may affect the traction performance and chatter instability [21,22], together with the *rear* and *front tire long* modes (due to tire tangential deflections).

As an example, Figure 5 depicts the open-loop eigenvalues of a naked motorcycle in straight motion for speeds from 10 to 50 m/s in 5th gear ratio (constant ratio has been used for simplicity, the engine spin rate ranging from 2000 to 9000 rpm). All the most important vibration modes are visible. In particular, in the range 25-30 Hz there is the first structural mode which is the front *fork bending*, in the 22-25 Hz there is *transmission* mode, in the 20-22 Hz there is *front tire long* mode, in the 10-15 Hz there are *front hop* and *rear hop* modes, at 7 Hz *rear tire long* and *wobble* mode, at 5 Hz *rider-shake* (lateral vibration of the rider on the saddle), then *weave* mode with frequencies rising from almost zero to 3.5 Hz (as speed increases), at 2 Hz *bounce* and on the x-axis *pitch* mode, and finally *capsize*, which is a non vibrating mode which involves the roll and the lateral displacement of the vehicle.

As a second example of frequency domain analysis, Figure 6 shows the FRF in the 0.1-1 Hz range from steering torque to roll angle of a street motorcycle performing a 14 m spacing slalom test. The comparison between experimental and computed FRF shows a good agreement.

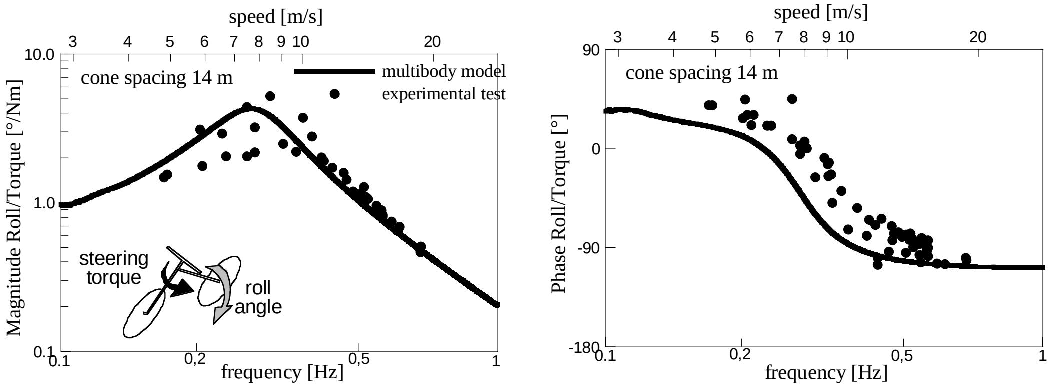

Figure 6. FRF from steering torque to roll angle of a street motorcycle on a 14 m spacing slalom.

3.3 Time Domain Analysis

As an example of time domain simulation, a lane change 4 x 21 (lane shift x lane distance) is performed at 18 m/s in Figure 7 and the Roll Index is computed [25]. A PID controller on the steering torque which observes the path error and the roll angle has been used for performing such a closed-loop manoeuvre. A proper set of gains has been computed by means of an optimization module included in the presented code, so that it is not necessary to go through a trial-and-error process to find the right gains. Further details on the implemented control architecture and optimization may be found in [4,8]. The vehicle handling while performing a lane-change may be evaluated by means of Roll Index (RI):

$$RI = \frac{\tau_{p-p}}{\dot{\varphi}_{p-p} V} = 1.35 \tag{6}$$

where τ_{p-p}(80 Nm) is the peak-to-peak value between the first two peaks of the steering torque, $\dot{\varphi}_{p-p}$ (3.3 rad/s) is the peak-to-peak value between the first two roll rate peaks, and V (18 m/s) is the vehicle average speed during the lane change.

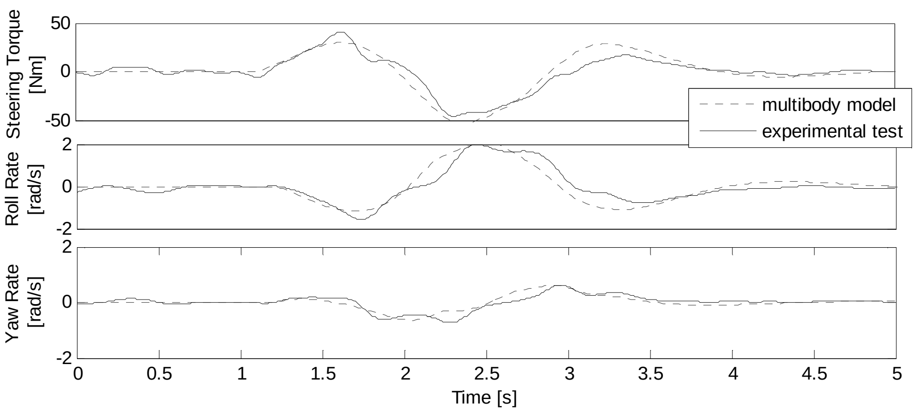

Figure 7. Lane change 4 x 21 at 18 m/s.

4 Conclusion

The latest improvement of the non-linear and fully parametric motorcycle multibody software developed at DIMEG have been described and the main modelling features illustrated: it exploits the advantages of the moving frame approach, includes both rigid and flexible bodies, a compliant transmission and a 3D road-tire interaction model.

The presented model carries out not only time domain simulation, but also steady state analyses (given the vehicle speed and accelerations) and the frequency domain analyses (eigenvalues and FRF): these are distinctive features and they are not easily available within general purpose multibody software.

Several simulation examples have been shown (steady state cornering, eigenvalues analysis, FRF from steering torque to roll angle, time domain lane change) and the significant handling indexes computed.

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